Combined Test questions – Spring 2011

Mat-221Spring 2011/ In class test # 1/ Halsey

Thursday, 3/3

Show all work and answers in the space provided. If you need additional room, please write on the back and indicate that I should look there.

1. (10 points) Show appropriate work that allows you to numerically estimate the derivative of \( f(x) = x^{(2x)} \) at \( x = 1.5 \). NOTE: The exponent MUST be in ( ). State what the derivative value is at \( x = 1.5 \) based on your tables.

   (4 points) Pick one value in the output column of your table, and show the function notation that was evaluated to produce this value, then explain verbally what this particular value represents.

   (4 points) Clearly explain what the tables above represent and how they help you “see” what the derivative for \( f(x) \) at \( x = 1.5 \)

2 (12 points) We have discussed derivatives of \( f(x) \) at \( x = a \) from a variety of perspectives. Write 3 complete ways to define the derivative of \( f(x) \) at \( x = a \) from a geometric, numerical and algebraic perspective WITHOUT telling me it is a real number, or an instantaneous rate of change.

   Geometric:
   Numerical:
   Algebraic:

3 (14 points) Algebraically compute the derivative for \( g(x) \) at \( x = 3 \) given that
\[
g(x) = x^2 - 3x + 2.
\]
Show ALL appropriate work and notation.

4 Given the data below about the derivative of a polynomial function \( h(x) \) and assuming it provides all crucial information

<table>
<thead>
<tr>
<th>( X_i )</th>
<th>( hprime )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-18</td>
</tr>
<tr>
<td>-5</td>
<td>0.8</td>
</tr>
<tr>
<td>-4</td>
<td>10.7</td>
</tr>
<tr>
<td>-3</td>
<td>13.5</td>
</tr>
<tr>
<td>-2</td>
<td>11.3</td>
</tr>
<tr>
<td>-1</td>
<td>6.2</td>
</tr>
<tr>
<td>0</td>
<td>-0</td>
</tr>
<tr>
<td>1</td>
<td>-5.2</td>
</tr>
<tr>
<td>2</td>
<td>-7.3</td>
</tr>
<tr>
<td>3</td>
<td>-4.5</td>
</tr>
<tr>
<td>4</td>
<td>5.3</td>
</tr>
<tr>
<td>5</td>
<td>24.2</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
</tr>
</tbody>
</table>

   a. (6 points) How many turning points does \( h(x) \) have? How do you know? Where are they?
   b. (6 points) How many points of inflection does \( h(x) \) have? How do you know? Where are they?
1. Given the graph $f(x)$ shown, assume that the turning points occur at $x = -3$ and $x = 2$. Also assume that the graph changes concavity at $x = -1/2$.

   a. (6 points) Explain how you can determine from the graph of $f(x)$ the interval(s) over which the derivative of $f(x)$ is decreasing, and show the interval(s) below.

   b. (6 points) Explain how you can determine from the graph of $f(x)$ the interval(s) over which the derivative of $f(x)$ is positive, and show these intervals below.

   c. (8 points) Sketch a clear graph of the derivative of $f(x)$, making it very clear where the graph crosses the x axis, and where the graph turns.

6. (12 points) Given the following information, first explain clearly what each piece tells you about $f(x)$, and then sketch a possible graph of $f(x)$.

   a. $f'(x) > 0$ for $x < -2$

   b. $f'(x) < 0$ for $-2 < x < 2$

   c. $f'(x) = 0$ for $x > 2$

   d. Sketch a possible graph of $f(x)$

7. (12 points) The temperature $T$ in degrees Fahrenheit, of a cold yam placed in a hot oven is given by $T = f(t)$, where $t$ is the time in minutes since the yam was put in the oven.

   a. What is the sign of $f'(t)$? Why?

   b. What are the units of $f'(t)$?

   c. Explain in contextual terms what $f(20) = 125$ means.

   d. Explain in contextual terms what $f'(20) = 2$ means.

Show all appropriate work and answers in the space provided. If you need more room, write on the back of the page, and indicate that I should look there.

1. (8 points) Find the equation of the line tangent to the graph of $y = x^2 + 6x$ at $x = 3$. Show all work clearly and completely.
2. (4 points each part) Given that \( H(3) = 1, \ H'(3) = 3, \ F(3) = 5, \ F'(3) = 4, \ F'(1) = 2 \) find \( G'(3) \) if
   a. \( G(x) = F(H(x)) \)
   b. \( G(x) = H(x) / F(x) \)

3. (12 points) Show clear work using derivatives to determine where the graph of \( y = x^4 - 4x^3 \) is concave up. Make it very clear what you are looking for and why.

4. (9 points each = 72 Points Total) Find the derivatives of the functions given using the rules we have covered in class. Simplify your answers, if appropriate, using the same models we have used in class.
   a. \( f(x) = 2x^{3/2} - 3x^{-1/3} + \cos(5x^2) \)
   b. \( h(x) = 3^{(-4x^2)} + 4\sin(x^3 + 6x) \)
   c. \( f(x) = (\sin^5 x)(\ln(x^2 - 2x)) \)
   d. \( f(x) = \sqrt{e^{-3x^2} + 5x} \)
   e. \( f(x) = \frac{e^{3x}}{x^4} \)
   f. \( f(x) = \cos(\ln(3x)) \)
   g. \( r(t) = \tan(2t + 3) \)
   h. \( f(x) = e^{\pi} + \pi^x + x^{2\pi} \)

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Mat 221/ Test # 3/ Spring 2011/ Halsey

Name __________________________

Show all work and answers in the space provided. If you need more room, you may write on the back of the sheets, but indicate clearly that I should look there.

1. (5 points each – 15 points) Find the derivative of each of the following. You do not need to simplify your answers.
   a. \( f(x) = \arcsin(x^2) \)
   b. \( f(t) = \sin(\arctan(3t)) \)
   c. \( f(x) = x^2 \arccos(x) \)
2. (12 points) Use appropriate Calculus ideas to find values of a and b so that the function 
\( f(x) = x^2 + ax + b \) has a local minimum at the point (4, -6). Make sure all essential work is 
clearly shown.

3. (12 points) Given the function \( f(x) = e^{-x^2} \), show appropriate work using derivatives to find 
the x values of any possible turning points and points of inflection. You DO NOT need to 
prove that the points you find are max, mins or points of inflection. You just need to find 
where such points could possibly occur.

4. (12 points) Find all critical points, and then use the first derivative test to determine local 
maxima and minima. Show work clearly, and completely, and clearly explain your final 
answer. \( f(x) = \frac{x}{x^2 + 1} \)

5. (10 points) Given \( f(x) = x^3 + 3x^2 - 9x - 15 \), find the critical points of the first derivative, 
and use the second derivative test to determine if you have max or mins. Show all essential 
work, and explain your final decision clearly.

6. (10 points) Find the global max and min for the function given on the given interval. Show all 
work clearly, and explain your decisions completely. \( f(x) = e^{-x} \sin x \) for \( 0 \leq x \leq 2\pi \)

7. For each of the following, show that you can use L’Hopital’s Rule, and then apply it to find the 
limit. All work must be clearly shown.
   a. (8 points) \( \lim_{x \to 1} \left( \frac{\ln(x)}{x^2 - 1} \right) \)
   b. (8 points) \( \lim_{x \to 0} \left( \frac{\cos(x) - 1}{2x^2} \right) \)

8. (12 points) Show clear work to find the dimensions given the minimum surface area given that 
the volume is 8 cubic cm for a closed rectangular box with a square bases with side x, and a 
height of h.